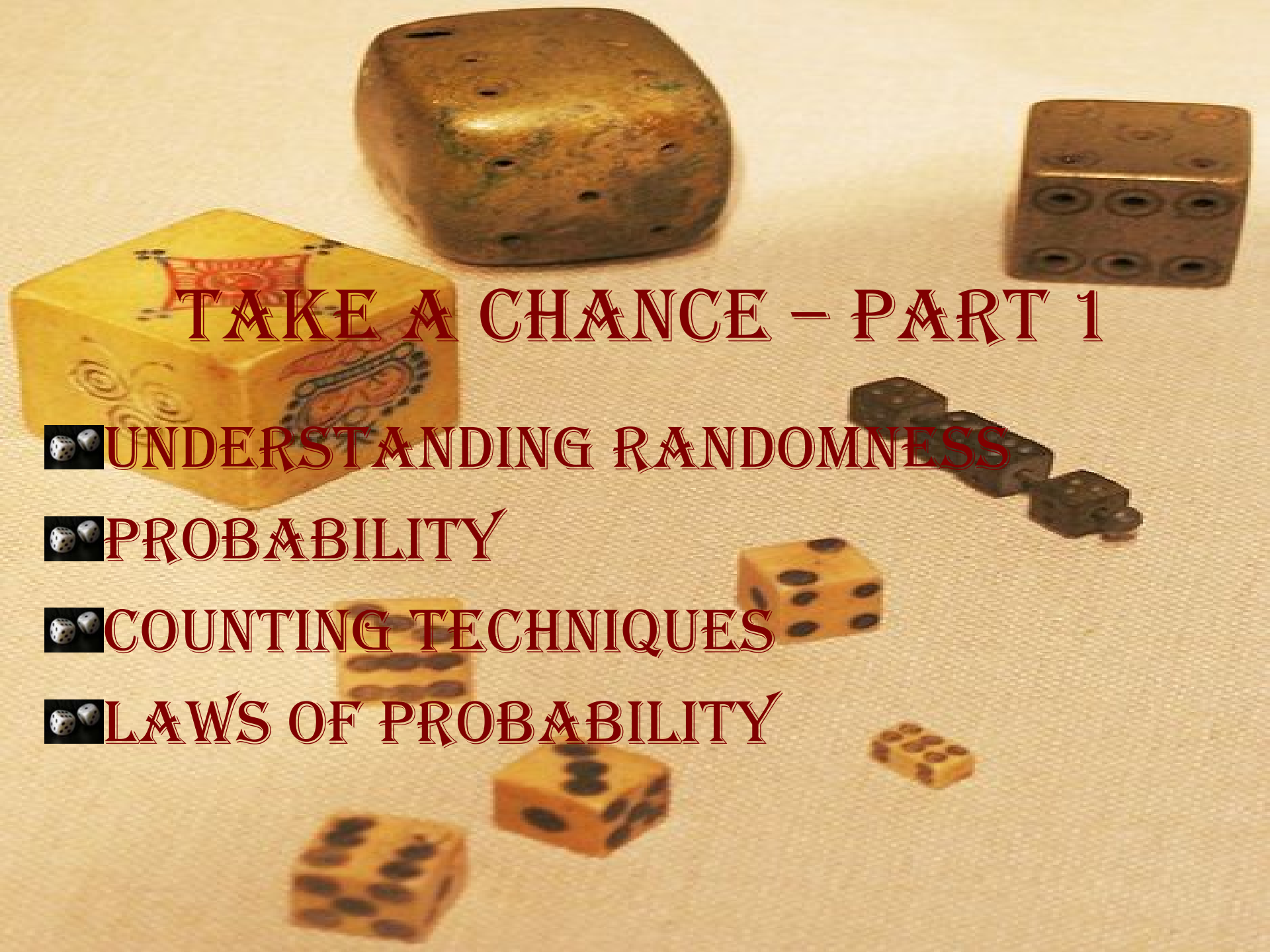
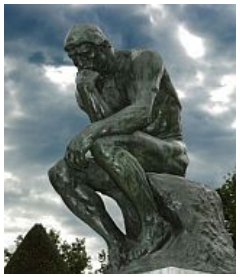


Take a Chance – Part 1

- Understanding Randomness
- Probability
- Counting Techniques
- Laws of Probability



Understanding Randomness

Random Activity – Thinking at Random

Flip mentally a quarter 16 times and record your results (H - for head, T - for tail)

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Why would you call the outcome random?

What is a "Random Event"?

 Random Activity – Acting at Random

Flip a real quarter 16times and record your results (H - for head, T - for tail)

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

In what major way is your result different from the mentally generated sequence ?

What is "wrong" (if anything) with your mentally generated result?

 Random Activity – Random Generations

OPEN http://www.random.org

CLICK **Games & Gambling:** Coin Flipper; SELECT 16 coins; SELECT your favorite coin

RECORD THE RESULTS

What does it look like, the hand-flipped or the mentally generated sequence?

Can a sequence be "Too random" and what could this mean?

Understanding Randomness

“…there are many … ways to get true randomness into your computer. A really good physical phenomenon to use is a radioactive source. The points in time at which a radioactive source decays are completely unpredictable, and they can quite easily be detected and fed into a computer, avoiding any buffering mechanisms in the operating system. The HotBits service at Fourmilab in Switzerland is an excellent example of a random number generator that uses this technique. Another suitable physical phenomenon is atmospheric noise, which is quite easy to pick up with a normal radio. This is the approach used by RANDOM.ORG. You could also use background noise from an office or laboratory, but you'll have to watch out for patterns. The fan from your computer might contribute to the background noise, and since the fan is a rotating device, chances are the noise it produces won't be as random as atmospheric noise. “*

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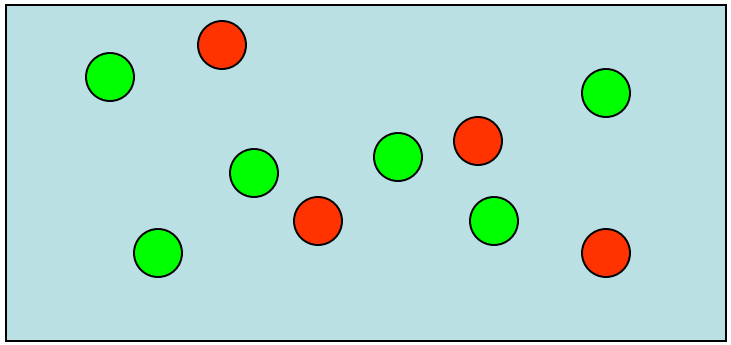
*) From: random.org

Probability

To calculate the probability of an event compute the ratio

$$\frac{\textit{Number of ways the event can be obtained}}{\textit{Total number of ways the experiment can be conducted}}$$

What is the probability of selecting a red marble out of a box containing 4 red and 6 green marbles?



Probability

What is the probability of getting a six when rolling a die?

$$\frac{\text{Number of ways the event can be obtained}}{\text{Total number of ways the experiment can be conducted}}$$

1	2	3	4	5	6

Probability

What is the probability of getting a sum greater than 7 when rolling two dice?

$$\frac{\textit{Number of ways the event can be obtained}}{\textit{Total number of ways the experiment can be conducted}}$$

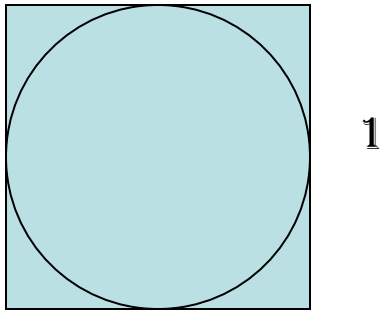
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability

What is the probability that a randomly thrown dart will hit the circle inscribed into 1 by 1 square?

$$\frac{\cancel{\text{Number of ways the event can be obtained}}}{\cancel{\text{Total number of ways the experiment can be conducted}}}$$

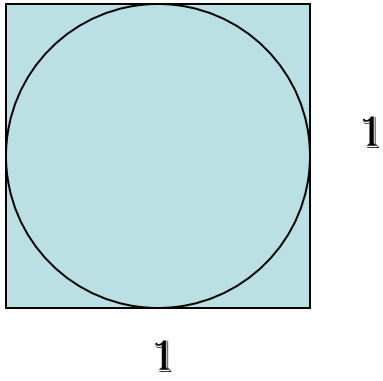
$$\frac{\text{Area of the circle}}{\text{Area of the square}}$$



Monte Carlo Method

Use Java Applet to estimate π

http://polymer.bu.edu/java/java/montepi/montepiapplet.html



Counting Techniques

Multiplicative Rule

Number of Outcomes for 1-st Event	x	Number of Outcomes for 2-nd Event	x etc =

Total Number of Outcomes

$$n = n_1 \times n_2 \times \ldots \times n_k$$

If you have 3 hats, 2 jackets, 6 shirts, 5 pairs of pants and 3 pairs of shoes.

How many different s outfits are possible?

Counting Techniques

Multiplicative Rule

Number of Outcomes for 1-st Event	x	Number of Outcomes for 2-nd Event	x etc =

Total Number of Outcomes

$$n = n_1 \times n_2 \times \ldots \times n_k$$

How many different plates are possible?

Permutations = rearrangements of distinct objects

Number of Objects for the 1-st spot	x	Number of Objects for the 2-nd spot	x etc =

Total Number of distinct sequences

- Read: n - factorial

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

$$0! = 1$$

In how many ways could the six bridesmaids be rearranged?

Permutations in a circle

Number of Objects for the 1-st spot	x	Number of Objects for the 2-nd spot	x etc =

Total Number of distinct sequences

Does not quite work (why?)

In how many ways could the five friends be rearranged?

Permutations = rearrangements of distinct objects

Number of Objects for the 1-st spot	x	Number of Objects for the 2-nd spot	x etc =

Total Number of distinct sequences

In how many ways could four bridesmaids be selected and rearranged?

Combinations = selections of *k* objects from *n* distinct objects

- Read: n – choose - k

$$n! = \binom{n}{k} k!(n-k)$$

so that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Lotto Game summary

Choose six Numbers from 1 to 49. Six Numbers are drawn.

Match all 6 Numbers to win the jackpot.

- In how many ways could the six winning Numbers be selected?
- What is the probability of winning the jackpot?

Combinations = selections of *k* objects from *n* distinct objects

- Read: n – choose - k

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Lotto Game summary
Choose six Numbers from 1 to 49. Six Numbers are drawn plus a bonus Number, out of 49 numbered balls. Match all 6 Numbers to win the jackpot. Match 5 Numbers and the bonus Number to win the second highest prize. Match five, four or three Numbers to win smaller prizes.

- In how many ways could the six winning Numbers and the bonus Number be selected?
- What is the probability of winning the jackpot or the second prize?

Putting the Counting Techniques to Work Together

BASEBALL LINE-UPS

Player rosters

Roster, or squad, sizes differ between different leagues and different levels of organized play. Major League Baseball teams maintain twenty-five-player active rosters. A typical twenty-five-man roster in a league without the without the DH rule, such as MLB's National League features:

- **five starting pitchers who constitute the team's starting rotation (careful! the order of players is important)**
- **eight position players—catcher, four infielders, three outfielders—who play on a regular basis**

Look at the Detroit Tigers active roster and count how many starting rotations are possible ?

Detroit Tigers Active Roster

#	Pitchers	B/T	Ht	Wt	DOB
38	Jeremy Bonderman	R-R	6'2“	220	Oct 28, 1982
49	Eddie Bonine	R-R	6'5“	220	Jun 6, 1981
40	Phil Coke	L-L	6'1“	210	Jul 19, 1982
59	Fu-Te Ni	L-L	6'0“	170	Nov 14, 1982
45	Ryan Perry	R-R	6'4“	200	Feb 13, 1987
48	Rick Porcello	R-R	6'5“	200	Dec 27, 1988
37	Max Scherzer	R-R	6'3“	220	Jul 27, 1984
36	Brad Thomas	L-L	6'4“	235	Oct 12, 1977
46	Jose Valverde	R-R	6'4“	255	Mar 24, 1978
35	Justin Verlander	R-R	6'5“	225	Feb 20, 1983
21	Dontrelle Willis	L-L	6'4“	225	Jan 12, 1982
54	Joel Zumaya	R-R	6'3“	210	Nov 9, 1984
#	**Catchers**	**B/T**	**Ht**	**Wt**	**DOB**
13	Alex Avila	L-R	5'11“	210	Jan 29, 1987
8	Gerald Laird	R-R	6'1“	225	Nov 13, 1979
#	**Infielders**	**B/T**	**Ht**	**Wt**	**DOB**
24	Miguel Cabrera	R-R	6'4“	240	Apr 18, 1983
4	Adam Everett	R-R	6'0“	180	Feb 5, 1977
15	Brandon Inge	R-R	5'11“	190	May 19, 1977
39	Ramon Santiago	S-R	5'11“	175	Aug 31, 1979
20	Scott Sizemore	R-R	6'0“	185	Jan 4, 1985
#	**Outfielders**	**B/T**	**Ht**	**Wt**	**DOB**
26	Brennan Boesch	L-L	6'4“	235	Apr 12, 1985
18	Johnny Damon	L-L	6'2“	205	Nov 5, 1973
14	Austin Jackson	R-R	6'1“	185	Feb 1, 1987
32	Don Kelly	L-R	6'4“	190	Feb 15, 1980
30	Magglio Ordonez	R-R	6'0“	215	Jan 28, 1974
25	Ryan Raburn	R-R	6'0“	185	Apr 17, 1981

Baseball Line-ups

Look at the Detroit Tigers active roster and count

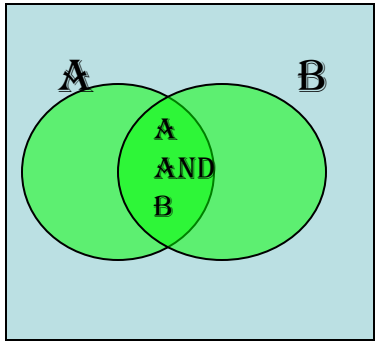
- In how many ways a catcher can be selected
- In how many ways the four infielders can be selected? How about the three outfielders?

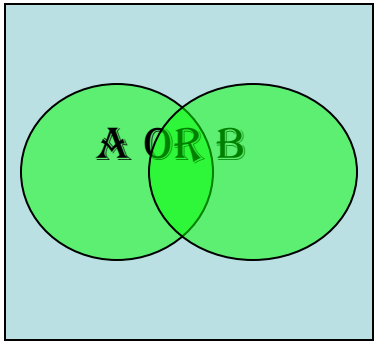
BASEBALL LINE-UPS

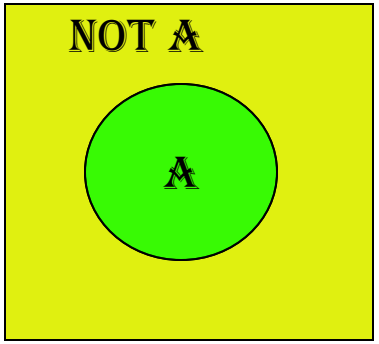
Look at the Detroit Tigers active roster and count

- In how many ways the four infielders can be lined-up? How about the three outfielders?
- How many different line-ups of an MLB baseball team are possible?

The Laws of Probability







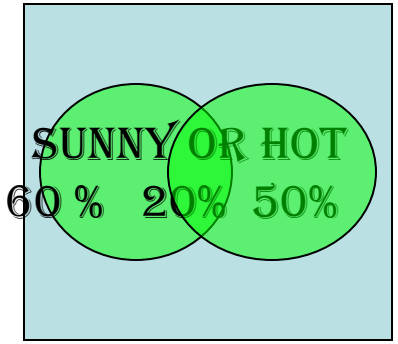
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(\text{Not } A) = 1 - P(A)$$

1. In one village, 60% of days are sunny, 50% of days are hot and 20% of days are sunny and hot.

   (a) What is the proportion of days that are sunny or hot?

   (b) That are not sunny?



A few interesting problems

1. All questions are about our group. Assume no leap years.

If we could choose our birthdays any way we wanted, in how many different ways could we do that?

Same question as above, but if everyone would have to have different birthday?

What is the probability that among a group of people like us at least two persons have the same birthday?

A few interesting problems

2. There are 3 letters and 3 envelopes.

In how many different ways could we put the letters inside the envelopes (one letter per envelope, of course)?

What is the probability that exactly one letter will end up in a wrong envelope?

What is the probability that exactly two letters will end up in wrong envelopes?

What is the probability that all three letters will end up in a wrong envelopes?

A few interesting problems

3. There are 23 letters and 23 envelopes.

In how many different ways could we put the letters inside the envelopes (one letter per envelope, of course)?

What is the probability that exactly one letter will end up in a wrong envelope?

What is the probability that exactly two letters will end up in wrong envelopes?

What is the probability that exactly three letters will end up in wrong envelopes?